doi: 10.1255/tosf.104

# The Pierre Gy Oration

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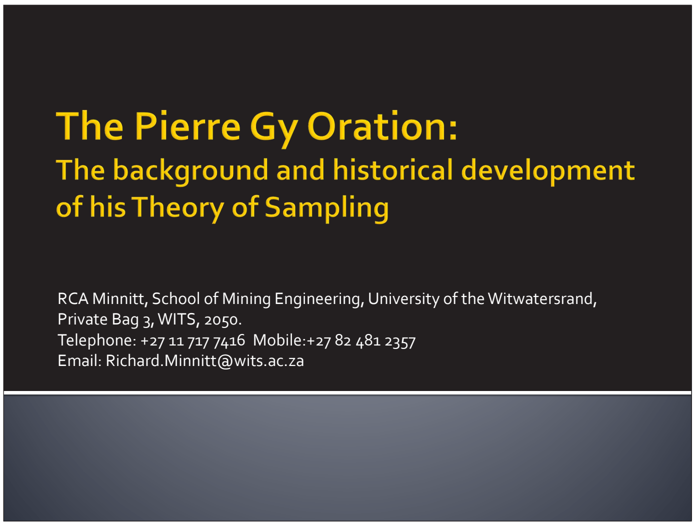
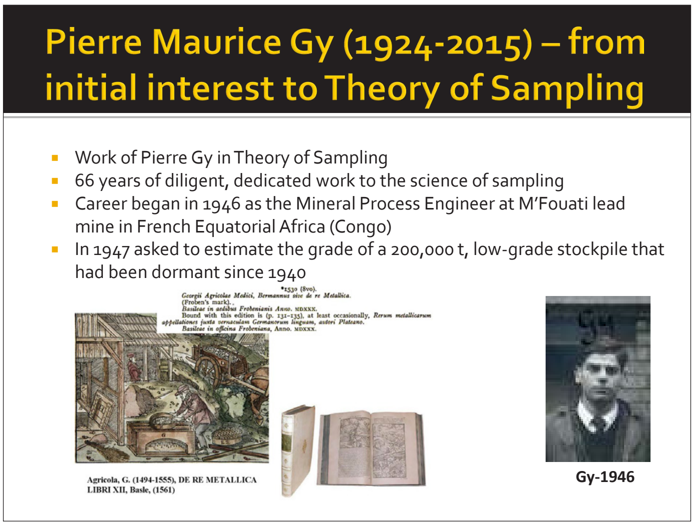
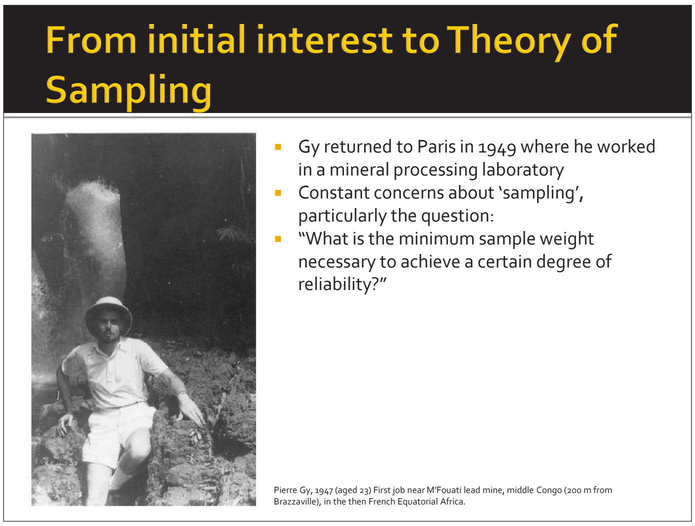
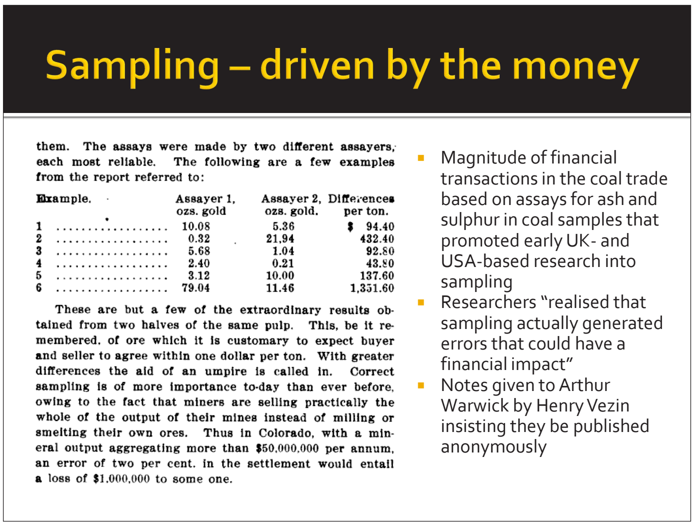
This is a graphic rendition of the inaugural "Pierre Gy Oration", a new institution affiliated with the Pierre Gy Sampling Gold Medal (PGSGM). The honour of being a recipient of the PGSGM is, henceforward, associated with the obligation to give this distinctive speech as the lead-in to the morning session after the WCSB gala dinner at which the medal is awarded. Richard Minnitt was the first to fulfil this honour. His presentation turned out to be a veritable feast for the eyes, in fact it was so enticing that it was decided to present it here in its original PowerPoint format, in order to give readers the most vivid impression possible. Enjoy Dick's comprehensive tribute to the founder of the Theory of Sampling.




them. The assays were made by two different assayers, each most reliable. The following are a few examples from the report referred to:

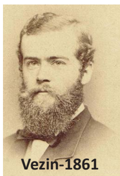
| Example. | Assayer 1, ozs. gold | Assayer 2, ozs. gold. | Differences per ton. |
|---|---|---|---|
| 1 .................. | 10.08 | 5.36 | $ 94.40 |
| 2 .................. | 0.32 | 21,94 | 432.40 |
| 3 .................. | 5.68 | 1.04 | 92.80 |
| 4 .................. | 2.40 | 0.21 | 43.80 |
| 5 .................. | 3.12 | 10.00 | 137.60 |
| 6 .................. | 79.04 | 11.46 | 1,351.60 |

These are but a few of the extraordinary results obtained from two halves of the same pulp. This, be it remembered, of ore which it is customary to expect buyer and seller to agree within one dollar per ton. With greater differences the aid of an umpire is called in. Correct sampling is of more importance to-day than ever before, owing to the fact that miners are selling practically the whole of the output of their mines instead of milling or smelting their own ores. Thus in Colorado, with a mineral output aggregating more than $50,000,000 per annum, an error of two per cent. in the settlement would entail a loss of $1,000,000 to some one.

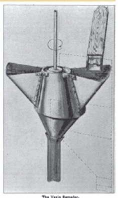
## Historical background

TAKING THE SAMPLE

- Henry Vezin (1866)
- Sylvanus Reed (1882/9)
- T. Clarkson (1893)
- David Brunton (1895)
- Heinrich Hofman (1899)
- P. A. Johnson (1902)
- Arthur Warwick (1903)
- Thomas Rickard (1905)
- Robert Richards (1907)
- John Church (1908)
- Bailey (1909)
- Philip Argall (1912)
- T. R. Woodbridge (1916)
- H.B. Pulsifer (1920)
- Arthur Taggart (1927)



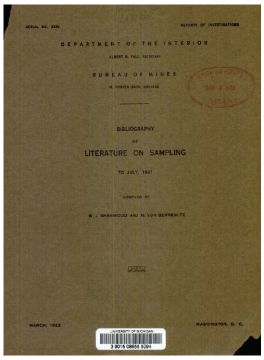
Vezin-1861



The Vezin Sampler.

Sampling and buying ore in the Joplin District by Evans Buskett, in The Engineering and Mining Journal, July 25th 1908. p190.

## Proliferation of work: 1890-1920 period



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H. FOSTER BAIN, DIRECTOR

BIBLIOGRAPHY

OF

LITERATURE ON SAMPLING

TO JULY, 1921

COMPILED BY

W. J. SHARWOOD AND M. VON BERNEWITZ

MARCH, 1922

WASHINGTON, D. C.



- There was a huge proliferation of literature at this time so that by 1921 Sharwood and Von Bernewitz through the US Bureau of Mines published a Bibliography of Literature on Sampling up to July 1921, that listed 906 articles, papers and books on the subject

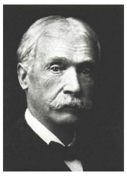
## Background

Fig. 2. OECC Mission (Europe), 1953. Meeting on: "The beneficiation of low-grade ores". As one of France's two delegates (aged 29, second from right), I am lazily listening to some lecture (the memory is not quite up to the photographic documentation; I have forgotten where the meeting actually took place. . .).

- He worked tirelessly, often in isolation, and not without resistance from other scientists
- His aim was to create a systematic, mathematically based framework within which every error arising from the sampling of a lot can be analysed, explained and named

## Early researchers

Figure 3. David William Brunton 1849-1927.[10] Courtesy of the Denver Public Library, Western History Department.



- Brunton (1895) – in order to relate particle size to sample mass, the minimum sample weight was proportional to the cube of the top particle size
- Gy's concern was that variations in grade or density had not been properly incorporated
- Robert Richards (1908) Prof of Mining Engineering at MIT, believed the cube of the particle size gave masses that were too high
- In order to comply with the belief that "the most satisfactory rule must be based on habits acknowledged by the trade of minerals" Richards arbitrarily used the square of the particle size

## Pierre Gy's early publications

- Gy's own mathematical model related variance of the sampling error to $M_L$, $M_S$, and physical properties of the material being sampled
- "The minimum sample mass for an acceptable sampling variance"
- No support in terms of time or resources for this research
- He devised a formula and the basic tenets of Theory of Sampling in two internal, unpublished notes for his company *Ste Minerais et Metaux*, entitled: "A formula for the minimum sample mass" and "Minimum sample mass required to represent a batch of ore" as early as 1950.

- P Gy (1953) "Erreur commise dans le prélèvement d'un échantillon sur un lot de minerai" Congres des Lavriesdes Mines Metalliques, Paris, September1953
- P Gy(1954) "Erreur commise dans le prélèvement d'un échantillon sur un lot de minerai" Congrès des laveries des mines métalliques françaises, Ecole des Mines de Paris Revue de l'IndustrieMinérale36 311 –345

## The Formula Established

- Progression in Gy's 1950's logic is fascinating
- He identified physically well-defined parameters:
  - $N_L$ the number of fragments in the lot
  - $N_S$ the number of fragments in the sample
  - $a_S$ grade of the sample
  - $F_i$ the total number of fragments
  - $M_i$ the average mass of fragments

- Strict algebraic relationships, simplified and approximated, for practical implementable formulae
- Mean and variance of population: "equally probable samples of $N_S$ fragments"
- Needed good understanding in statistics – therefore earned a second PhD

## Gy's equations

- Heterogeneity - most important concept, "lies at the root of all sampling errors"

$$h_i = \frac{(a_i - a_L)}{a_L} \times \frac{M_i}{M_i^*}$$

  - $h_i$ - constitutional heterogeneity carried by one fragment $F_i$ in the lot $L$
  - $M^*_i$ - average mass of all individual fragments $F_i$
  - $h_i$ – contribution to heterogeneity carried by each fragment in the lot
- Which when converted to an average variance $h_i^2/N_L$ for the lot and divided by the number of fragments in the sample $N_S$ given an appropriate statistical weight
- Approximate measure of variance of Total Sampling Error, $^2_{TSE}$

$$\sigma^2_{TSE} = \left( \frac{1}{N_S} - \frac{1}{N_L} \right) \cdot \frac{\sum_{i=1}^{N_L} h_i^2}{N_L}$$

## The Formula

- Trials-and-errors Gy tested many simplifications and approximations
- Arrived at the well-known general equation

$$\sigma^2_{FSE} =\sim \left( \frac{1}{M_S} - \frac{1}{M_L} \right) \cdot cfg\ell d^3$$

$$=\sim \frac{cfg\ell d^3}{M_S} \quad when\ M_S << M_L$$

- Referred to as "Gy's formula" , but he preferred 'The Formula'
- First presented in English to the Society of Mining Engineers of the American Institute of Mining Engineers (SME of AIME) in 1957
- Only in 1965 that his research was presented in London at a meeting of the Institution of Mining and Metallurgy (IMM).

## Parameters for the Formula

- c - mineralogical composition factor; takes average grade and density of all components into account specific gravity (g/cm³)
- c = /$a_S$ -increases as average grade decreases
- $\ell$ - dimensionless liberation factor defined by Francois-Bongarcon as
- $\ell = (d_\ell/d_N)^{0.5}$ varies betwen 0 (liberated) to 1 (non-liberated)
- $f$ - dimensionless particle shape factor, 0.5
- $g$ - dimensionless size range factor, 0.25
- $d$ - top particle size (cm) passing 95% of material

## Probabilistic sampling model

- A highly simplified version of the probabilistic model is shown below:

$$\sigma^2_{FSE} = \frac{1-P}{PM_L} \cdot CH_L$$

$$= \left[ \frac{1}{M_S} - \frac{1}{M_L} \right] HI_L$$

Where the Heterogeneity Invariant: $HI_L = cfg\ell d^3$, and hence

$$\sigma^2_{FSE} = \left[ \frac{1}{M_S} - \frac{1}{M_L} \right] \cdot cfg\ell d^3$$

50th wedding anniversary with wife Sylvia, daughter Caroline and grandson Stanislas.

## Extensions to the model

- His interest in sampling led to a first theoretical model for sampling of heterogeneous lots made up of particulate solids
- Then developed a generalised model for solids of animal and vegetable origin, types of domestic and industrial waste, liquids and gasses

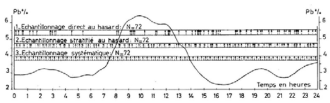
- By this stage Gy recognised that the models had universal validity and that it was *scale* rather than physical state that differentiated between the range of applications.

Pierre Gy, extreme left, was 30 years old when this picture was taken in 1954.

## Validating the Formula

- In validating his formula from the variance of sixteen equally split samples of pulverised lead ore, he found experimental errors larger than theoretical TSE:
- He therefore concluded that the Fundamental Sampling Error – is only one of several components

- Other components were GSE and sampling bias from incorrect use of riffle splitter
- Mid-1950's development of a circular cardboard sampling nomogram and later a sampling slide rule

## Sampling of flowing streams

- Pierre Gy's 1960-1962 research into flowing streams of materials on conveyor belts and liquid launders
- Worked on velocity of cutters in cross-stream sampler, the width of the cutter opening and the shape of the cutter
- Gy recognised that increments extracted at constant intervals from a flowing stream are not independent from one another, there is some level of *auto-correlation* between time series sample data
- In 1962 Gy published work on *chronostatistics*, linear auto-correlation of time series data, borrowing the concepts of semi-variograms proposed by Matheron and later David
- Early '60's Gy chose writing and research of ToS over the comfort of Minerais et Metaux in Paris
- This began a grand forty-year period of theoretical research, consulting, lecturing and teaching regular courses

## A period of opposition

- Serious opposition from members of the scientific communities 1967 French publication, "Sampling of Particulate Materials"
- ISO response was less than accepting of Gy's work
- His 1971 book entitled "Sampling of Particulate Materials, Volume 2" was soon followed by another book "The Theory and Practice of the Sampling of Particulate Materials" in 1975, but only a few hundred copies were ever sold
- Pierre Gy (2004a) tells of the difficulties he faced in 1978-1979 writing his first book in English, a translation of the 1975 text. The book, published in 1979, was followed by a second edition in 1982.
- Between the release of the Second Edition of the 1979 text and his latest book in French (1988), Gy developed a number of new applications of his theory including the computation of auxiliary functions of the variogram, the ideas underlying proportional sampling, and a theory of bed-blending

## Sample selection – sampling errors

- Notion of "correct sampling" and its linkages to probabilistic sampling were first proposed by Gy in 1972.
- Concept in principle known by others (c.f. Vezin), Gy was the first to articulate sampling correctness "if, and only if, each and every fragment has the same statistical probability of being selected for the sample, as every other fragment".
- Gy built "the mathematical bridge between practical selecting conditions and theoretical sampling errors"
- Identified distinction between *a priori* conditions of sample selection, conditions we can do something about before taking the sample, and *posteriori* conditions of sampling selection, conditions we observe, but about which we can do very little after the fact
- Selection process: probabilistic or non-probabilistic, and even if probabilistic; it *could* be correct or incorrect.

## Proportional Sampling

- Metallurgical balance reconciliation in a North African lead-zinc flotation plant
- "whatever comes in must ultimately come out, one way or another" with a single exception in 45 years, everything that ever came out was always less than what went
- Principal culprit for 2-3% deficit was calibration of conveyor belt scales
- Conveyor belt scales suffer from a structural lack of reliability, - converting electrical current to a measurement of ore tonnes
- Proportional sampler to metallurgical balances


Pierre Gy in 1975, just turned 50

## Proportional sampler

- Gy concluded that if the probability of selection $P$ is a uniform distribution, then sampling is correct, and the mean of the sample mass $M_S$ is a random variable equal to $P$ times the mass of the lot $M_L$

$$m(M_S) = P \cdot M_L$$

- Accurate estimation of $P$ means that $M_S/P$ is an unbiased estimator of the mass of the lot, $M_L$
- All concentrates, tailings and feed streams, sampled according to the same selection probability make the proportionality factor constant.
- $M_S/P$ is a more reliable estimator of mass of lot $M_L$, than any that can be obtained by weightometers and is the basis for idea of *proportional sampling*
- Gy defined a time sampling ratio and a mass sampling ratio

## Derivation of Proportional Sampling

| Explanation | Equation | Parameters |
|---|---|---|
| "time sampling ratio" of a lot, ' | $\tau' = \frac{QT_i}{T_L}$ | $T_L$ = flow time of the lot L<br>Q = No of increments, t=0 to t=$T_L$<br>$T_i$ = time to take one increment |
| "mass sampling ratio" of a lot, | $\tau = \frac{M_S}{M_L}$ | $M_S$ = mass of the sample S<br>$M_L$ = mass of the lot L |
| If sampling is correct | $\tau' = m(\tau)$ | m( ) is the mean of |
| Rearranging | $\tau' = \frac{m(M_S)}{M_L}$ | $M_L = \frac{m(M_S)}{\tau'}$ |

Sampling from all material streams is comparable, making the calculation of the material balance a simple task.

## Bed blending

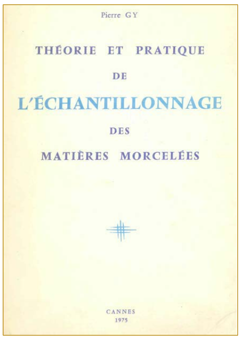
- Raw material feed to metallurgical furnace must be more-or-less uniform and homogenous
- Gy's bed blending studies began with input to cement kilns
- Costly damage to cement kilns can be avoided using uniform feeds
- Bed blending systems used to homogenise raw materials feed
- Good sampling equipment allowed major element analyses every few minutes
- Computerised assistance to calculate the average composition of the stockpiled kiln feed allowed the composition of the blending pile to be known with accuracy, providing an almost ideal feed to the kiln

## Bed blending

- Gy found that bed blending theory was easily derived from existing sampling theory
- The manufacturer received an excellent explanation of how his equipment worked
- Gy's bed blending theory was in perfect agreement with practice and proved attractive to other cement producers
- Theory and practical aspects published in 1981 with a presentation to Canadian Institute of Mining and Metallurgy (CIM) in Montreal
- Theories that Gy published over the years have consistently proved to be correct, and were easily adapted to the science of bed blending.

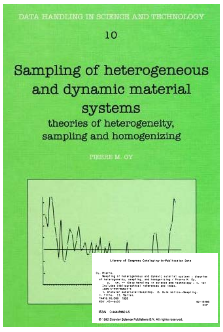
https://upload.wikimedia.org/wikipedia/commons/thumb/e/e9/KilnBZ.JPG/250px-KilnBZ.JPG

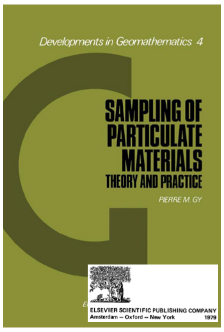
## Pierre Gy's Publications

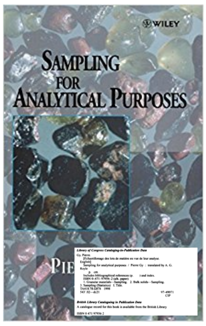



Pierre Gy with wife Sylvia at the WCSB1 banquet, Esbjerg, Denmark August 2003.

## Pierre Gy's Publications



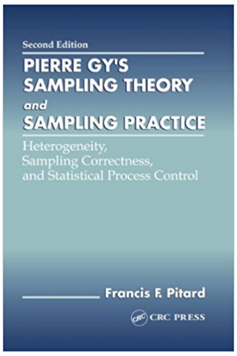




## Francis F Pitard's English translation

- Industry standard has become the 1992 publication:
- "Pierre Gy's Sampling Theory and Sampling Practice,

Heterogeneity, Sampling Correctness and Statistical Process Control"

by Francis F Pitard




## The legacy

- WCSB1 - Esbjerg, Denmark
- WCSB2 - Brisbane, Australia
- WCSB3 – Porto Alegre, Brazil
- WCSB4 – Cape Town, South Africa
- WCSB5 – Santiago, Chile
- WCSB6 – Lima, Peru
- WCSB7 – Bordeaux, France
- WCSB8 –Perth, Australia

- **TOS Forum**, Communication forum for the theory and practice of representative sampling (TOS) community

- **International Pierre Gy Sampling Association (IPGSA) (2017)**