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"Critique of Gy's Sampling Theory": Misplaced expectations of Wikipedia's democratic intentions

Geoffrey J. Lyman and Kim H. Esbensen

In today's age of the internet and the cloud's many "blessings", Wikipedia is widely hailed as the pre-eminent internet source of readily available information. Wikipedia has especially been acclaimed for its apparent *democratic* attitude towards building a free, open encyclopaedia of the time. Indeed Wikipedia carries a plethora of truly informative entries, and there are but few who have not had reason to sample information from this source. But there is also a darker side to all this enthusiasm—in that *anybody* can enter *any* new entry where none exists on a given topic, or edit any existing article. In fact, upon reflection, it dawns upon users that this democratic openness is not necessarily a blessing. Thus this institution has aptly been described by the following depressing characterisation: "Wikipedia is the medium in which your worst enemy can get to write your epitaph". This statement can act also as a clear pointer to our errand here regarding a contribution to Wikipedia in which a number of faults and accusations levelled at the Theory of Sampling (TOS) and its proponents unfortunately can be found. We find it incumbent upon us to draw public attention to this criticism of the entire life-time's achievement of Pierre Gy and the Theory of Sampling (TOS).

TOS critique in Wikipedia

We recently were directed to the fact that an entry is included in Wikipedia under the title "Gy's Sampling Theory",[1] in which a number of faults in the theory are implied. The Wikipedia text also provides a reference to an open access viXra.org (http://www.vixra.org/abs/1203.0081) document authored by Dihalu and Geelhoed. These two contributions are critical of Gy's work, and a full assessment of all scientific aspects with which the present authors, indeed most of the TOS community, will take issue will be presented elsewhere.

Suffice here to point out that Geelhoed has previously presented a paper that sought to question the matter of quantifying sampling variance in the presence of non-independent particle selection probabilities. This issue is at the root of Geelhoed's criticisms, and has also been published in several other fora. Geelhoed's work, as reported at the Third World Sampling and Blending Conference (WCSB3), Porto Alegre, is based on a new mathematical simulation approach to predicting sampling variance but provided no experimental results. This work harks back to his paper to the sampling community, presented at WCSB2, Brisbane, which did contain some experimental results and where the math behind the proposed new approach was first put forward. However, the experimental work was only directed at extremely simplistic two-component systems of particles with slightly different sizes (but identical composition and hence density), from which sweeping conclusions were attempted that claimed to represent inherent deficiencies in the foundation of Pierre Gy's Theory of Sampling. These claims, and especially their foundation, have been criticised on several occasions by several of the leading members of the sampling community.

First and foremost, it must be understood that the entire critique exclusively only addresses issues related to estimating the Fundamental Sampling Error (FSE) and that all Geelhoed's work only relates to Pierre Gy's 1979 work,[2] but nowhere refers to the three most fundamental works in the context, viz. Gy papers in 1967 and 1971,[3–5] which rank among the most central works specifically describing the issues surrounding the genesis of FSE—and the realisation of strict limits for the realistic application of the equation for its estimation. It has been pointed out to both Geelhoed and Dihalu on various occasions in several fora, that several empirical results and experiences from extensive experimental campaigns led Pierre Gy himself to conclude that the possibilities for the simple, first order "Gy's formula"[a] are more limited than many practitioners would like to accept, limited except for rather simple systems. Pierre Gy concluded that a second (of the so-called "correct sampling errors") was needed, the Grouping and Segregation Error (GSE), if one was ever to get a realistic grasp of the full complexity of the phenomena of heterogeneity. It is fair to state that this insight has been pointed out to Geolhoed *et al.*, but to no apparent avail, and this is especially germane to the entry in Wikipedia. With this background, we here focus on a few salient issues in the "critique".

It appears that the critical focus point in Geelhoed's assertions is that the random selection of a particle of one type to fall into an increment (a sample) may *influence* the selection probability for the following particle (a physical neighbour particle). That is, it is proposed that the selection probability for the second particle is not independent of the selection of the previous particle. This then might be the case where a "type 2" particle tends to associate with a "type 1" particle. This situation is well known from TOS as the case of "spatial coherence" or "grouping" if occurring in a broadly isotropic material, and as "segregation" in the case where such a tendency to coherence is primarily brought about by gravitation. In fact these relationships were discussed extensively in the (1967, 1971) fundamental Gy literature.[3–5] These issues are of course also present in any-and-all of Gy's later

[a]Gy himself loathed that this equation has been accorded this personal accolade—by others, who are not necessarily initiated to the full complexity of heterogeneity and how to counteract this in sampling. Gy has in fact always been highly dissatisfied and worried that his name should be associated with "just a first attempt, and a simplistic and highly approximate equation at that— trying to encapsulate something much more complex" (pers. com. 2008). This personal insight is key to understanding much of our vehement rejection of the Wikipedia "critique".

publications; a complete bibliography of Pierre Gy can be found in the proceedings WCSB1.[6]

Being not unskilled in statistical matters, and having reviewed Geelhoed's WCSB2 paper, one of us (GL) was hard pressed to understand the manner in which Geelhoed arrived at his final equations and conclusions. In response, he examined the problem of non-independent particle selection probabilities using a similar Markov process approach as Geelhoed and concluded, quite opposite to Geelhoed, that the non-uniform selection probability had only little impact on the sampling variance. This counter paper was also presented at WCSB3, Porto Alegre. It is fair to say that the scientific opposition that ensued here did not lead to substantial changes in either position.

It is a fact that Gy's work has sought only to identify first order effects on sampling variance, indeed Gy himself was adamant in pointing this out. Much of his early work was directed at elucidating the theoretical relationship associated with what became known as the Fundamental Sampling Error only (expressed both in theory and in practise as a variance). In analysing a given sampling circumstance, all experienced samplers, experts and consultants in the field have to work with *approximate data*, not with perfect statistical distributions. In all realistic situations in the field, in the plant or in the laboratory, in general one does not have access to full knowledge such as the distribution of particles' grades with respect to the critical analyte on a size by size basis, which is at the root of dealing theoretically with FSE, and which is the necessary foundation for simulating a sampling process. Nor does one possess full knowledge of the covariance function for grade in the process stream being analysed for example, another characteristic that needs to be known in order that realistic simulations can even begin to be contemplated. And finally, crucially, no one has the necessary means, short of prohibitively expensive experimentation, of assessing, for example, the extent of lateral particle segregation by grade on a conveyor belt from which a sample (or an increment) is to be drawn. It is absolutely critical to understand, and acknowledge, that such extra-FSE heterogeneity *per force* will change from second to second, minute to minute, hour to hour, day to day in the course of events of realistic sampling of real-world lots and materials—such is the nature of significantly heterogeneous materials. Instead one works with summary information obtained from preliminary, pilot study *heterogeneity tests* on material that has been collected with the specific objective of being representative of the general material class to be sampled, now **and** in the future, over some time-span of the task or project at hand. In this work, one relies, for example, on variograms estimated from survey samples collected from a process stream under conditions that are carefully characterised and which must be representative of the future sampling process. Above all one strives to the utmost to make the sampling process(es) "correct", i.e. unbiased, the conditions for which forms the most important part of TOS and which must be included in all types of evaluations of a realistic sampling process. To take one example, the efficiency of a sampling process is based on analysis of samples that span the full relevant range of compositional variations to be encountered in future applications of the same sampling process, either to a similar class of material and/or to similar material in the future. Armed with this type of empirical data relevant for the materials and processes at hand, one *may* now evaluate with some reliability the likely magnitude of the sampling variance that will be encountered. Based upon this kind of knowledge, one may venture further to design sampling systems that will then achieve, on average, a level of precision that is deemed to be economically important to the operation—after the accuracy issue (the bias) has been first eliminated by designing, and implementing "correct" sampling procedures. Here one seeks to provide a mechanically correct sampling system to ensure that the total remaining sampling uncertainties are controlled to an acceptable level. If one can deliver this, then and only then, the professional sampling job has been done.

All the above is a far cry from the conditions that underlie the simulations reported by the works referred to in the Wikipedia entry—indeed the simulations covered by these references can only be characterised as extremely simplistic—without any realistic relevance except for a simplistic case of an ideal two-component system. Geelhoed has been carefully informed of the extreme deviation between this situation and real-world heterogeneity on many occasions (KHE).

Pierre Gy himself, after extensive experimental work carried out to test the realism of FSE estimates, realised and publically acknowledged the existence and significance of such extra-FSE heterogeneity in the overwhelming majority of materials. For this reason he conceptualised and coined the second of the so-called correct sampling errors, the Grouping and Segregation Error (GSE), aimed at representing the sampling variance effects stemming from this irregular meso- to macro-scale heterogeneity characterising the lot geometry realm *beyond* the scale of one particle and its retinue of surrounding secondary particles. In this Gy was very much aware that the simple statistical apparatus he had used to start analysing the relationships regarding FSE would only be able to further a first order approximation. This is a demonstrable fact in several key publications from 1967 onwards (referred to in Reference 7). So, Gy was very well aware that in the realm to which he assigned the GSE, matters could not be subjected to any then-known statistical treatment. He would, however, undoubtedly, have welcomed any such professional attempt, as should all subsequent sampling theoreticians and practitioners for that matter. And this is precisely the realm to which Geelhoed and Dihalu direct their attention, indeed the PhD thesis of Dihalu bears the intriguing title: "The *Terra Incognito* of Sampling: Grouping and Segregation".[8] However, the Wikipedia entry and the open access document referenced contain severe misunderstandings of the nature of TOS, and, in our view unacceptably disrespectful comments are levelled at the intentions of its originator. We can only take up the most blatant such issue here.

"Fudge factors"

First, the criticism of the two parameters introduced in TOS to achieve a more detailed description of heterogeneity, the grouping- and the segregation parameters. These are directly called "fudge factors" (also in a few places in Dihalu's PhD thesis).

In attempting to estimate the influence of segregation in the body of a mineral mixture, we are essentially blind without

truly exhaustive sampling work, which is generally unjustified and anyway quite prohibitive in everyday sampling. Francis Pitard has said of the influence of segregation that: "If quantified today, it will differ tomorrow". This is a truism that has not been recognised by the authors of the critique. Central to our rejection hereof, and again an issue repeatedly presented to Geelhoed (KHE): Pierre Gy's grouping and segregation parameters are of a **totally different nature** in TOS: these are presented in the theoretical analysis of the complex concept of heterogeneity as "phenomenological parameters", intended to represent the influence from grouping ("groups-of-particles, clumpiness") and segregation in the formal mathematical–statistical apparatus developed by Gy for this purpose. This simply could not be further away from the postulated nature of "fudge factors" stated by Geelhoed and Dihalu, e.g. documented by a direct quote from their open access document: "The use of fudge factors to tweak the predicted values with the experimental values is a major point of concern in Gy's theory". TOS' phenomenological factors were never intended to be estimated and used to bridge the gap between the formula-based (FSE only) and empirical estimates (FSE + GSE). This is a fatal misunderstanding. For want of space, we refer the reader to the scholarly treatment of these issues in much more theoretical and practical depth (which indeed also is a direct response to the TOS criticism delivered by Geelhoed at three WCSB conferences) given by Pitard & Bongarcon.[7] The denigration of Gy's theoretical work as depending on fudge factors is an insult to all serious scientific dealings with the Theory of Sampling.

Overview of contrasting positions

To claim that Gy's theory needs rectification, on the basis of data collected under the particular circumstances of simplistic mixing is a red herring cast across the path of the use of Gy's work. The present authors have both been using Gy's results and methods since 1982 (GL) and 2000 (KHE), the former as a consultant with extensive experience from many industry sectors and application fields, the latter heavily involved in teaching and dissemination of TOS (also including many industrial sectors, corporate and regulatory bodies), and we have found no fault with the theory and application at all. If Geelhoed and Dihalu wish to construct a revised theory of sampling, this is a fully legitimate objective, and indeed one that would only meet with approval by all parties. But (the absolutely central issue), anybody undertaking such an endeavour **must** per force provide cogent descriptions of the alternative theory and back it up with solid, very careful and extensive experimentation. Most of all, it is incumbent upon any such contenders to provide **evidence** (theory and experiment) that the new theory provides results that make a **significant** difference to Gy's results. If the differences are only small, if the issues only address FSE, and if all issues related to the bias-generating incorrect sampling errors are totally **ignored**, all of which pertain to the Wikipedia "critique", one need absolutely not abandon Gy's work. Geelhoed should rather find means and ways to provide sound and full theoretical coverage as well as realistic experimental evidence, that Gy's work is in significant error before continuing to denigrate this work. In our experience, TOS has, throughout all of its 60 years' of existence, firmly defended all tests of theoretical riguour and practicality, over and over. There is an overwhelming published, peer-reviewed literature to back this up.

Conclusion

It is not wrong to (try to) level criticism at the Theory of Sampling (TOS). TOS is no sacred object. TOS is a comprehensive, indeed claimed to be a complete, theory of heterogeneity, sampling and the derived principles for design of representative sampling procedures and equipment. Nobody in the TOS community would object to the continued testing of theoretical concepts, or to assessment and evaluation of the practical correspondence with reality of TOS. Indeed this takes place all the time, as can be followed in full public detail in the continuing series of WCSB proceedings.

What is wrong, and what has lead to our strong consternation and rejection of the Wikipedia "Critique of TOS" section, is the superficiality in the levelled critique which represents a total lack of respect for the entire life's work, the formidable *oeuvre* of Pierre Gy. This will simply not stand.

Appropriate measures to have the current entry removed from Wikipedia and replaced with a more fitting, scientifically sound and more respectful entry is under way.

The reader is encouraged to make her/himself acquainted with the Wikipedia entries and to form their own opinion. Readers are invited to join in this endeavour, either by voicing their dissatisfaction with the current entry, or by presenting their reasons for supporting the Geelhoed & Dihalu claims. *TOS forum* is open to all reactions to the issues raised above.

References

1. https://en.wikipedia.org/wiki/Gy%27s_sampling_theory (accessed 29 October 2013).
2. P.M. Gy, *Sampling of Particulate Materials, Theory and Practice. Developments in Geomathematics 4*. Elsevier Scientific Publishing Company (1979).
3. P. Gy, "L'chantillonage des Minerais en Vrac. Volume 1", *Revue de l'industri Minérale*, St Etienne, France (Numéro Spécial) (15 janvier 1967).
4. P. Gy, "Modele General de l' Echantillonnage des Minerais (General Model for Sampling Mineral Ores)", *Revue de l'Industrie Minerale* **53(2),** 77–99 (1971).
5. P. Gy, "L'chantillonage des Minerais en Vrac. Volume 2", *Revue de l'industri Minérale*, St Etienne, France (Numéro Spécial) (15 septembre 1971).
6. K.H. Esbensen and P. Minkkinen (Eds), "Special Issue: 50 years of Pierre Gy's Theory of Sampling. Proceedings: First World Conference on Sampling and Blending (WCSB1). Tutorials on Sampling: Theory and Practise", *Chemometr. Intell. Lab.* **74(1),** 1–236 (2004).
7. F.P. Pitard and D. Francois-Bongarcon, "Demystifying the Fundamental Sampling Error and the Grouping and Segregation Error for practitioners", in *Proceedings: World Conference on Sampling and Blending, WCSB5*, Santiago, Chile, 25–28 October 2011. GECAMIN, pp. 39–56 (2011). ISBN: 978-956-8504-59-5
8. D.S. Dihalu, *The* Terra Incognito *of Sampling: Grouping and Segregation*. PhD thesis, Technische Universiteit Delft. Publisher IOS Press, The Netherlands (2012). ISBN: 978-1-61499-074-1