Heterogeneity Characterization for Sampling Variance Prediction

By Dominique François-Bongarçon[1]

DOI: 10.62178/sst.003.005

1. Gy's Formula and The Liberation Factor

As mentioned in a more comprehensive contribution at WCSB11 (Francois-Bongarcon, 2024), the analytical model proposed by Gy for *fully liberated* materials was as follows:

$$\text{Rel.Var.} = cfgd^3_{95}(1/M_S - 1/M_L) \quad (1)$$

(for liberated, comminuted, or naturally occurring particulate material)

In this formula, only some parameters can be set at the sampling time: the sample mass Ms, and the comminution P_{95} size d_{95}. Other parameters can be calculated from known properties of the material to be sampled: the mineralogical constant c, the shape factor f, and the granulometric factor g.

The restriction of the formula to fully liberated materials was an obvious impediment to using it for practical predictions of sampling variances in general cases. Gy therefore offered a modified version:

$$\text{Rel.Var.} = c\,\ell\,fgd_{95}^3\,(1/M_S - 1/M_L) \quad (2)$$

in which a ‚liberation factor', ℓ (numerical values between 0 and 3) was introduced to account for the degree of non-liberation of the material. No acceptable and practical working model was offered for this factor ℓ until the present author's work in the 90's.

To conform with Gy's intuitions about ℓ being directly correlated to the proportion of liberated material and to additional De Wijsian geostatistical considerations, the fractal, heuristic model then proposed:

$$\ell = (d_\ell / d)^b \quad (3)$$

introduced two more parameters in formula (2):

- a De Wijsian exponent, b, which is linked to the model of clustering of the pure analyte particles in the gangue, and
- a particle size parameter d_ℓ often likened to the analyte 'particle liberation size'.

Heterogeneity characterization studies aim at finding the best values for these parameters to successfully customize formula (2) for the material at hand. A comprehensive overview of the critical conceptual assumptions and practical technical issues that must be observed was given by Chieregati (2024).

THE STUDY OF HETEROGENEITY

Pierre Gys' legacy includes not only the most impressive synthesis of the first principles that will guarantee successful use of sampling for a variety of applications, but also a numerical model aimed at helping the practitioner accurately predict sampling precision from sampling parameters. Naturally, this step implies a customization of the analytical model of variance, often known as 'Gy's formula'. This customization is often coined 'heterogeneity study', and is always aimed at predictions and the solving of what-if scenarios.

1 Agoratek International Consultants Inc., Canada.

2. A Simple, Fundamental Remark

The analytical simplicity of these models is deceptive, but their calibration is a critical success factor of great importance. Indeed, a poor calibration, especially of the De Wijsian exponent, may easily result in grossly optimistic, or pessimistic, variance predictions. In the first case, enormous amounts of money may be lost in the long run depending on the application (e.g., grade control in a mine). In the second, huge capital money may be wasted (e.g. over-dimensioning of sampling preparation laboratories). The importance of properly performing heterogeneity characterizations cannot be under-estimated. Ad hoc methods, simplifications and all-made recipes are dangerous and should be avoided for this delicate purpose.

This said, that importance is often obfuscated by false debates between specialists. While some, with us, do advocate carefully performed heterogeneity studies using proper, non-liberated models, other think they are either invalid in their essence, too complex to be performed well, or even useless. This author strongly believes these discussions to be irrelevant, because they invariably take place between practitioners who reason in completely different contexts and with non-comparable objectives: characterizing a given situation versus predicting what-if scenarios.

In the case of an open pit mine with blast hole sampling, the basis of grade control, for instance, very distinct problems can be considered.

- The sole determination of the minimum sample mass to reach a desired precision does not require the full models of formulas (2) and (3) above and instead can be solved with a simple sampling experiment.
- Conversely, the dimensioning and capital optimization of a sample preparation laboratory to process the samples once they are collected, would clearly imply the prediction of a variety of possible sub-sampling stages scenarios, with several hypothetical variations in comminution sizes (and therefore in states of liberation), rendering necessary the use of a model for the liberation factor.

Finally, humility is required. One cannot lightly criticize models that, properly applied, have successfully received Georges Matheron's criterion of the 'sanction of practice' over a period of more than 30 years (Matheron, 1989).

3. Recommendations

After describing common heterogeneity characterization practices, the above-referenced WCSB11 paper lists a series of Do's and Don'ts to help the practitioner perform meaningful calibrations. We only need to repeat/enhance them here, to make the present short communication a flag-waiving referral companion to the 2024 WCSB11 paper.

3.1 Do's

- Perform a careful analysis of all available data: are they representative of the sampling case that needs to be performed?
- Try to determine which type of material (e.g., mineralization type) gives the *worst* response to sampling – then focus on this case.
- Try to objectively eliminate outlying data without letting that operation bias the final results. There is no doubt that this issue demands the largest possible experience: Don't do this on you own, if you are not competent – Do contact experienced colleagues or consultants.
- Understand fully the issue at stake, especially the economic consequences of the heterogeneity calculations, and the most critical aspects of their applications.
- Take your time: these are delicate empirical operations; they need to be performed with an intimate understanding of what needs to be properly achieved.
- Formula (2) is for a single stage of sampling, and not, as would the case be for the variance calculated from the grades of routine samples, the variance of a full series of cascading sampling operations alternated with comminution stages. Therefore, when using one of the methods involving splitting of a series of samples, make sure the experiment is properly designed so that the variances calculated from laboratory assay results can be 'cleaned up' before being equated to formula (2). In other words, one must be able to remove unwanted variance components from the results (such as those due to subsequent preparation and sub-sampling on top of the primary sampling operation). This, and only this, will allow to correctly equate the sampling formula (2) to the result of a resulting single-stage, primary sampling operation. In particular, in a simple case where the samples in one series are only pulverized and assayed after being collected, then removing the pulp sampling and analytical variances from the total assay variance requires that one of the available series be a series of samples taken from already pulverized material.

Not removing that component will always give a high-biased value of De Wijsian exponent b, with an enormous effect on predicted sampling variances.

- The splitting methods used to generate a series of samples *should* give random samples reasonably protected from the effects of natural segregation. Riffle splitting, alternate shoveling and fragment-per-fragment selection are the recommended methods. Rotary splitters, on the other hand, do not give random samples and can therefore not be used to calibrate a theoretical formula they have nothing to do with. When this difference was discussed with Pierre Gy, he told the author of this paper that the difference between random and rotary sampling was akin to the difference between dealing a thoroughly shuffled deck of cards (riffling or random sampling) and to doing so after carefully sorting the deck by color and card values (rotary splitting). Indeed, to the usual surprise of many a practitioner, that method (rotary splitting), a circular version of bed-blending, uses segregation as an advantage. Any segregation aggregates that pass through the system is falling into the containers of the carousel in such a way that it is uniformly distributed in them, a feat that cannot be achieved by random sampling of any kind. Segregation in that process, contrary to the case of random sample selection (e.g., in random increments); therefore, is an additional, favorable feature for the division. Thus, segregation is used to the advantage of representativeness, to such a point that the corresponding sampling errors have variances potentially much lower than those of regular random sampling, i.e., lower than predicted by TOS formulas.
- If the calibration uses a graphical approach, care should be taken to make sure the quantities plotted together on the same calibration graph were made directly comparable/compatible (i.e., representing the same quantities as a function of the abscissa). To achieve this, instead of plotting only the 'Rel.Var.' quantity of formula (2) as a function of d95, the plotted quantity should absorb any factor of difference. For instance, if two points on the graph correspond to: i) sampling of closely sieved material, and ii) sampling a full-size distribution of material, then the plotted quantity should be first divided by the respective granulometric factors g and g'. If various points have different grades, the quantity should also be divided by 'c' (which is a function of grade) before plotting it.

3.2 Don'ts

- First of all, don't fall victim to ready-made, ad hoc formulas and nomograms that were published in the past (more particularly pre-1992), they simply will not work.
- Don't over-trust QA/QC duplicate sample results. Such duplicate samples were manifestly not collected for this purpose, and they are often consciously or unconsciously *doctored*, by removing parts of what needs to be quantified. Indeed, it is standard and normal for a laboratory manager to review the assays before they are delivered to the laboratory's client. However, any duplicate result deemed abnormal in his/her own judgement or intuition (eventually ill-informed) will be removed, factored or redone.
- Be keenly aware of the differences between *sampling* (TOS) and *measuring* (geostatistics), between samples and measurement supports and therefore do not use measurement support duplicates (i.e., repeats of some in-situ sampling, such as *duplicate* channel samples or *duplicate 1/2 core* as a mere examples).
- Of course, the samples should be able to represent TOS formula (2) for random samples, which precludes using non-random splitters such as rotary ones.
- The parameters obtained should ultimately be compatible with the sampling procedures to design, diagnose, or optimize. In particular, sampling characteristics of a lot of comminuted material simply cannot be derived for a single size-fraction, which, alone, may easily have different sampling properties than the whole.

4. Conclusion

While heterogeneity studies may appear somewhat complex at first sight, they are not ‚rocket science' – but they must be performed very carefully with full acknowledgement of the specific experimental conditions, as their economic impact can be devastating if they are strongly biased. The present author has treated the topics covered above on several occasions since 1992. More comprehensive analysis, argumentation, and documentation can be found in the bibliography below.

References

François-Bongarçon, D. (2024). Heterogeneity Studies: Do's and Don'ts. *The Proceedings the World Conference on Sampling and Blending 11*, May 21-23, Misty Hills Conference Center, Muldersdrift, Johannesburg, South Africa, The Institute of Mining and Metallurgy (SAIMM), pp. 15-24. ISBN: 978-1-7764673-0-3, https://www.saimm.co.za/Conferences/files/wcsb11-2024/02_WCSB649-Francois-Bongarcon.pdf

Chieregati, A.C. (2024, Nov.). A Brief Review of Heterogeneity Tests for Estimating the Variance of the Fundamental Sampling Error. *Sampling Science & Technology Magazine*, 2, pp.19-29, DOI: 10.62178/sst.002.003, https://www.sst-magazine.info/wp-content/uploads/2024/11/sst-002-003.pdf

Matheron, G. (1989). *Estimating and Choosing: An Essay on Probability in Practice*, A.M. Hafsofer (trans.), Springer-Verlag, Berlin, New York, London, Paris. DOI:10.1007/978-3-642-48817-7

Bibliography

François-Bongarçon, D. (1992). Geostatistical Tools for the Determination of Fundamental Sampling Variances and Minimum Sample Masses, *Proceedings of Geostatistics Troia ,92. Kluwer Academic Pub.* 2, Troia, Portugal, pp. 989-1000.

François-Bongarçon, D. (2008). Sampling and Heterogeneity Studies Creative Methods. *Proceedings of W Sampling Conference*, 27-29 May, Perth, W. AUS., Victoria, AUS: Australasian Institute of Mining and Metallurgy (AusIMM) pp.67-68, ISBN: 978 1 920806 81 1

François-Bongarçon, D. (2017). The evolution of the concept of liberation factor and a surprising new result. In S. Dominy & K.H. Esbensen (Eds.), *Proceedings of Eighth World Conference on Sampling and Blending*, 9-11 May, Perth, W.AUS., Victoria, AUS.: Australasian Institute of Mining and Metallurgy (AusIMM), pp. 307-310, ISBN: 978 1 925100 56 3

François-Bongarçon, D. (2022). Liberation factor for ‚closely sieved' material. In K. H. Esbensen & A.J. Cole (Eds.), *Proceedings of Tenth World Conference on Sampling and Blending*, 31 May - 2 June, Kristiansand, NO., *TOS forum 11*, Chichester, UK.: IMpublication Open LLP, pp. 7-9. doi:10.1255/tosf.xxx

François-Bongarçon, D. & Gy, P. (2002). The Most Common Error in Applying ‚Gy's Formula' in the Theory of Mineral Sampling, and the History of the Liberation Factor. *The Journal of The South African Institute of Mining and Metallurgy*, Nov./Dec. https://www.saimm.co.za/Journal/v102n08p475.pdf

François-Bongarçon, D. & Pitard, F. (2015). Comparison between samples with constant mass and samples with constant fragment population size, In *Proceedings of Seventh World Conference on Sampling and Blending*, K. H. Esbensen & C. Wagner (Eds.), 10-12 June, Bordeaux, FR. *TOS forum 5*, Chichester, UK.: IMpublication Open LLP, pp. 231-238. ISSN: 2053-9681.